HW #22: Worksheet on Section 3.4 KEY

1. f is concave up on $(-2,2)$ since $f''>0$ and f is concave down on $(-\infty,-2)\cup(2,\infty)$ since $f''<0$. $x=2$ is a point of inflection since f'' changes signs from positive to negative and $x=-2$ is a point of inflection since f'' changes signs from negative to positive.

2. f is concave up on $(2,\infty)$ since $f''>0$ and f is concave down on $(-\infty,0)\cup(0,2)$ since $f''<0$. $x=2$ is a point of inflection because f'' changes signs from negative to positive.

3. f is concave up on $(-\infty,2)\cup(4,\infty)$ since $f''>0$ and f is concave down on $(2,4)$ since $f''<0$. $x=2$ is a point of inflection since f'' changes signs from positive to negative and $x=4$ is a point of inflection since f'' changes signs from negative to positive

4. f is concave up on $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ since $f''>0$ and f is concave down on $\left(0,\frac{\pi}{2}\right)\cup\left(\frac{3\pi}{2},2\pi\right)$ since $f''<0$. $x=\frac{\pi}{2}$ is a point of inflection since f'' changes signs from negative to positive and $x=\frac{3\pi}{2}$ is a point of inflection since f'' changes from positive to negative.

5. f has a local maximum at $x=-4$ since $f'(-4)=0$ and $f''(-4)=-14$.

6. f has a local minimum at $x=\frac{\pi}{6}$ since $f'\left(\frac{\pi}{6}\right)=0$ and $f''\left(\frac{\pi}{6}\right)=1$.

7. Crit. pts: $x=0,2$

$x=0$ is a relative maximum because $f'(0)=0$ and $f''(0)=-6$.

$x=2$ is a relative minimum because $f'(2)=0$ and $f''(2)=6$.

8. Crit. pts: $x=\pm 2$

$x=-2$ is a relative maximum because $f'(-2)=0$ and $f''(-2)=-1$.

$x=2$ is a relative minimum because $f'(2)=0$ and $f''(2)=1$.

9. Crit. pts: $x=\frac{3\pi}{4},\frac{7\pi}{4}$

$x=\frac{3\pi}{4}$ is a relative maximum because $f'\left(\frac{3\pi}{4}\right)=0$ and $f''\left(\frac{3\pi}{4}\right)=-\sqrt{2}$.

$x=\frac{7\pi}{4}$ is a relative minimum because $f'\left(\frac{7\pi}{4}\right)=0$ and $f''\left(\frac{7\pi}{4}\right)=\sqrt{2}$.

10. 2004 AB 4/BC 4 See AP Central for full solution.

(a) Show with implicit diff.

(b) $y = 2$

(c) $\frac{d^2y}{dx^2}=-\frac{2}{7}$. At (3, 2), $\frac{dy}{dx}=0$ and $\frac{d^2y}{dx^2}=-\frac{2}{7}$ so the curve has a local maximum at (3, 2) by the Second Derivative Test.

11. (a) f is increasing on $(-\infty, 0)$ and $(3, \infty)$ because $f'(x)>0$.

f is decreasing on $(0, 3)$ because $f'(x)<0$.

(b) f has a relative maximum at $x = 0$ because $f'(x)$ changes from positive to negative there.

f has a relative minimum at $x = 3$ because $f'(x)$ changes from negative to positive there.

12. (a) f is decreasing on $(-\infty, -1)$ and $(3, 5)$ because $f'(x) < 0$.

f is increasing on $(-1, 3)$ and $(5, \infty)$ because $f'(x) > 0$.

(b) f has a relative minimum at $x = -1$ and $x = 5$ because $f'(x)$ changes from negative to positive there.

f has a relative maximum at $x = 3$ because $f'(x)$ changes from positive to negative there.

13. f has an inflection point at $x = 1$ and at $x = 7$ because $f''(x) = 0$ and $f''(x)$ changes from positive to negative or vice versa there. f does not have an inflection point at $x = 4$ because $f''(x)$ does not change signs at $x = 4$.

14. $a = 6$, $b = 9$

15.

Point	f	f'	f''
A	+	+	–
B	+	0	–
C	–	–	+